Total No. of printed pages = 8

END SEMESTER EXAMINATION – 2022

Semester : 1st

Branch : Common

Subject Code : Sc-102

MATHEMATICS – I

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Instruction :

• *All* questions of PART-A and PART-B are compulsory.

PART – A

Marks – 25

1. Choose the correct answers of the following : $1 \times 10 = 10$

(i) The value of i^{29} is

(a) $-i$ (b) i

(c) -1 (d) 1

(ii) The value of $\cos 1^0 \cos 2^0 \cos 3^0 \_\_\_\_ \cos 100^0$ is

(a) 1 (b) -1

(c) 0 (d) $\frac{1}{2}$

(iii) The argument of $2+2i$ is

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) 1

(iv) Number of terms in the expansion of $(x+a)^n$ is

(a) $n+1$ (b) $n-1$

(c) n (d) n^2

(v) The value of $\log_2 \log_3 81$ is

(a) 2 (b) 4

(c) 0 (d) 3

(vi) The value of $\sin(-120^0)$ is

(a) 0 (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) $-\frac{\sqrt{3}}{2}$

(vii) In how many ways can the letters of the word COLLEGE be arranged ?

(a) 5040

(b) 2520

(c) 1260

(d) 5038

(viii) The value of $\sin^2 48 + \sin^2 42$ is

(a) 0 (b) 1

(c) -1 (d) 2

(ix) The value of $\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}$ is

(a) π (b) 0

(c) $\frac{\pi}{2}$ (d) 1

(x) The greatest angle of the triangle whose sides measure 3cm, 4cm and 5cm is

(a) 45^0 (b) 90^0

(c) 60^0 (d) 30^0

2. Wtite true or false : $1\times5=5$

(i) The partial fraction of $\frac{2x-3}{(x+2)(x+4)}$ is $\frac{-7}{4(x+2)}+\frac{11}{2(x+4)}$.

(ii) If two rows of a determinant are identical then the value of the determinant is zero.

(iii) The condition of perpendicularity of the lines in terms of their slopes is $m_1m_2=-1$.

(iv) In any triangle ABC, $c = a\cos B + b\cos A$.

(v) $\cos 2\theta = 1+2\sin^2\theta$.

3. Fill in the blanks : $1\times10=10$

(i) The value of 6_{p3} is _____.

(ii) Area of regular hexagon of side 'a' is _____.

(iii) The surface area of a sphere is _____.

(iv) If $A=\begin{bmatrix}3 & 2\\ 1 & 5\end{bmatrix}$ then 3A is _____.

(v) Value of $\cot(\tan^{-1}a+\cot^{-1}a)$ is _____.

(vi) The volume of a cylinder having radius of the base 3cm and height 4cm is _____.

(vii) The general term in $\left(x+\frac{1}{x}\right)^n$ is _____.

(viii) The order of the matrix $\begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$ is _____.

(ix) The x-intercept of $3x+4y+5=0$ is _____.

(x) Area of a triangle with base 'b' and height 'h' is _____.

PART – B

Marks – 45

4. Answer any *five* of the following questions : $2\times5=10$

(i) Prove that $(1+\omega)(1+\omega^2)(1+\omega^4)(1+w^8)=1$, where ω is the imaginary cube root of unity.

(ii) Prove that $7\log\frac{10}{9}-2\log\frac{25}{24}+3\log\frac{81}{80}=\log 2$.

(iii) If $A^T=\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A+2B)^T$.

(iv) Find n if $n+2_{p3} = 5\times n+1_{p2}$.

(v) Find the term independent of x in $\left(2x+\frac{1}{3x^2}\right)^9$.

(vi) If $A=\begin{bmatrix}3 & 1\\ 4 & 1\\ -5 & 2\end{bmatrix}$ and $B=\begin{bmatrix}3 & 1 & 2\\ -2 & 0 & 4\end{bmatrix}$ then find the value of AB.

5. Resolve into partial fractions (any *one*) :$3\times1=3$

(a) $\frac{x^2-3x+1}{(x-1)^2(x-2)}$.

(b) $\frac{x+29}{x^2+8x-9}$.

6. Answer the following questions : $3\times2=6$

(a) if a, b, c be the p^{th}, q^{th} and r^{th} terms of an AP, prove that $a(q-r)+b(r-p)+c(p-q)=0$.

(b) Prove that

$$\begin{vmatrix}1 & a & a^3\\ 1 & b & b^3\\ 1 & c & c^3\end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

Or

Solve by Cramer's rule $5x-y=9$

$3x+y=7$

$x+y+z=4$.

7. Answer any *three* of the following questions : $2\times3=6$

(a) $\dfrac{\sin A+\sin B}{\cos A+\cos B}=\tan\dfrac{A+B}{2}$.

(b) Prove that, $3\tan^{-1}x=\tan^{-1}\dfrac{3x-x^3}{1-3x^2}$.

(c) Show that $\cos(60^0-A)\cos(30^0-B)-\sin(60^0-A)\sin(30^0-B)=\sin(A+B)$.

(d) Prove that,

$a\sin(B-C)+b\sin(C-A)+c\sin(A-B)=0$.

8. Answer any *two* of the following questions : $3\times2=6$

(a) If $A+B=45^0$, prove that $(1+\tan A)(1+\tan B)=2$.

(b) If $A+B+C=\pi$, prove that

$\sin^2A+\sin^2B+\sin^2C=2+2\cos A\cos B\cos C$.

(c) If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, prove that $x+y+z=xyz$.

9. Find by Simpson's rule the area of the curvilinear figure whose ordinates measure 18, 22, 26, 24, 20, 26, 30, 34, 28, 24, 14 metres and whose base is 150 metres. 3

10. Find the whole surface area of a right prism whose height is 75cm and whose base is a regular octagon of ride 12cm. 3

11. Answer the following questions : 2+3+3=8

(a) Find the distance of the point (3,–5) from the line $3x-4y-26=0$.

(b) Find the equation to the line passing through (1, 2) and parallel to $6x+2y-3=0$.

(c) Find the angle between the lines $7x-y=1$ and $6x-y=11$.